

# AN ENGINEERING METHOD FOR COMPUTING THE DYNAMIC CHARACTERISTICS OF CASE-TUBE HEATERS

M. Ya. Khait

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An analytical procedure for computing the dynamic characteristics of steam case-tube heaters is proposed. The technique is illustrated by an example.

The dynamic characteristics of heaters are computed by constructing partial differential equations describing the convective heat transfer process between the heating steam and product separated by a conductive partition in the heater.

An analytical expression for the dynamic characteristics can be obtained by simultaneous solution of the differential equations constructed for the non-steady-state heat transfer process.

We make the following assumptions in order to simplify our mathematical investigation:

1) We can assume developed turbulent flow of the fluid, as a result of which the temperature of the product can be considered uniform over the pipe cross section; this enables us to allow only for the temperature variation in the direction of flow (the point of entry into the heater is taken as the origin);

2) the specific heats remain constant;

3) the density of the product is constant;

4) the mechanical energy of the product (i.e., its kinetic and potential energies) is negligible as compared with its thermal energy;

5) we can neglect the accumulating capability of the heat exchanger wall; as is shown in [1], such an assumption is legitimate in calculating heat exchange equipment and can be regarded as the first approximation of the actual process. This entails replacing the heat exchange coefficients in the differential equations by heat transfer coefficients, and the wall temperatures by the temperatures of the heat carriers flowing on the opposite side;

6) the convective heat transfer coefficients are practically independent of the expenditure and temperature of the heat carriers;

7) ideal mixing of the heat transfer agent takes place inside the case.

## DETERMINING CAPACITANCE FROM THE STEAM SIDE

The material balance equation for the steam space for a constant level of the condensate is

$$G_1 = G_2 + V_1 \frac{d\gamma_1}{dt} \quad (1)$$

Assuming that the temperature of the condensate is equal to the steam temperature  $T_1$  [2, 3], we can write the thermal balance equation for the steam space when

the steam feed rate changes by the amount  $G_1$  in the time  $dt$  as

$$G_1 i'' = G_2 i' + V_1 \frac{d(\gamma_1 U_V)}{dt} + G_{20} c_c T_1 + C_3 \frac{dT_1}{dt} + F_1 k_0 (T_1 - T_2) + Q_1, \quad (2)$$

where  $V_1 d(\gamma_1 U_V)$  is the energy accumulated in the volume  $V_1$  in the time  $dt$  (the energy of expansion or compression of the steam).

We assume, moreover, that the temperature of the outside wall of the steam jacket is equal to the temperature of the steam in the jacket; this is allowed for by the term  $C_3 dT_1$  in Eq. (2).

Substituting the value of  $G_2$  from (1) into expression (2), recalling that  $G_{10} = G_{20}$  in the steady state, and assuming that the thermal losses are constant, we write

$$G_1 (i'' - T_{10}) = -V_1 T_{10} \frac{d\gamma_1}{dt} + V_1 \frac{d(\gamma_1 U_V)}{dt} + C_3 \frac{dT_1}{dt} + G_{10} T_1 + F_1 k_0 (T_1 - T_2). \quad (3)$$

For small deviations of the variables we have

$$\gamma_1 = \gamma_{1m} + \alpha T_1; \quad U_V = U_m + \beta T_1. \quad (4)$$

Since intense mixing occurs in the case, the parameters of the steam space vary with respect to time only, i.e.,  $\partial T_1 / \partial x = 0$  and  $(\partial(\gamma_1 U_V) / \partial x) = 0$ .

Expression (3) therefore becomes

$$G_1 (i'' - T_{10}) = \gamma \frac{dT_1}{dt} + G_{10} T_1 + F_1 k_0 (T_1 - T_2), \quad (5)$$

where

$$\gamma = [V_1 (U_m - T_{10}) \alpha + V_1 \gamma_{1m} \beta + C_3]. \quad (6)$$

The steam feed rate through the heater  $G_{10}$  depends on the degree of opening of the regulating valve  $\xi_0$ , its specific permeability  $C$ , and the pressure drop across the valve  $(P_m - P_{10})$  [4]:

$$G_{10} = 31.6 \epsilon_{ef} \xi_0 C \sqrt{\gamma_{10}} \sqrt{P_m - P_{10}}. \quad (7)$$

When the valve is opened by a further amount  $\Delta \xi$ , the pressure in the heater case changes by  $\Delta P_1$  and the steam expenditure changes by the amount  $G_1$ .

Assuming that the steam pressure in the main is constant, we have

$$G_{10} + G_1 = 31.6 \epsilon_{ef} (\xi_0 + \Delta \xi) C \sqrt{\gamma_{10}} \sqrt{P_m - P_{10} - \Delta P_1}. \quad (8)$$

Replacing  $\sqrt{P_m - P_{10} - \Delta P_1}$  by its approximate expression, subtracting (7) from (8), and omitting terms of the second order of smallness, we obtain

$$G_1 = 31.6 \varepsilon_{ef} C \sqrt{\gamma_{10}} \sqrt{P_m - P_{10}} \Delta \xi - \frac{31.6 \varepsilon_{ef} \xi_0 C \sqrt{\gamma_{10}} \Delta P_1}{2 \sqrt{P_m - P_{10}}} \quad (9)$$

Introducing the notation

$$\begin{aligned} \text{a) } R_1 &= \frac{1}{F_1 k_0}; \quad r_0 = i'' - T_{10}, \\ \text{b) } a &= 31.6 \varepsilon_{ef} C \sqrt{\gamma_{10}} \sqrt{P_m - P_{10}}, \\ \text{c) } a' &= \frac{31.6 \varepsilon_{ef} C \sqrt{\gamma_{10}} \xi_0 \eta}{2 \sqrt{P_m - P_{10}}} \end{aligned} \quad (10)$$

and taking account of the small parameter variations  $\Delta P_1 = \eta T_1$ , we obtain from expression (9) the following dynamics equation for the steam space:

$$a r_0 \Delta \xi = \gamma \frac{dT_1}{dt} + (G_{10} + a' r_0) T_1 + \frac{1}{R_1} (T_1 - T_2) \quad (11)$$

#### DETERMINING THE CAPACITANCE FROM THE PRODUCT SIDE

The thermal balance equation for an incompressible fluid characterizing the heat flux per element of length of the product conduit can be constructed on the basis of the following considerations.

The heat influx per volume element is

$$\frac{F_1 k_0}{fL} (T_1 - T_2) \quad (12)$$

This heat increases the product temperature at the rate

$$\frac{dT_2}{dt} = \frac{\partial T_2}{\partial t} + v \frac{\partial T_2}{\partial x} \quad (13)$$

The thermal balance equation for a volume element of the product is

$$\frac{\partial T_2}{\partial t} + v \frac{\partial T_2}{\partial x} = \frac{1}{C_1 R_1} (T_1 - T_2) \quad (14)$$

where

$$C_1 = fL \gamma_2 c_1 \quad (15)$$

#### SUM EQUATION OF THE HEATER

From (11) and (14) we obtain

$$\begin{aligned} R_1 \gamma \frac{dT_1}{dt} + R_1 (G_{10} + a' r_0) T_1 + \\ + (T_1 - T_2) = R_1 r_0 a \Delta \xi, \\ C_1 R_1 \frac{\partial T_2}{\partial t} + C_1 R_1 v \frac{\partial T_2}{\partial x} = T_1 - T_2 \end{aligned} \quad (16)$$

Let us introduce the notation

$$\begin{aligned} \text{a) } a_1 &= \gamma R_1; \quad a_2 = C_1 R_1, \\ \text{b) } a_3 &= R_1 (G_{10} + a' r_0); \quad a_4 = R_1 r_0 a. \end{aligned} \quad (17)$$

System (16) then becomes

$$\begin{aligned} a_1 \frac{\partial T_1(t)}{\partial t} + (a_3 + 1) T_1(t) - T_2(x, t) = a_4 \Delta \xi(t), \\ a_2 \frac{\partial T_2(x, t)}{\partial t} - T_1(t) + T_2(x, t) = -a_2 v \frac{\partial T_2(x, t)}{\partial x} \end{aligned} \quad (18)$$

The initial conditions for the heater under consideration (for variable component temperatures) for  $t = 0$  and  $x > 0$  are

$$\begin{aligned} T_1(0) = 0; \quad \frac{\partial T_1(0)}{\partial t} = 0; \quad T_2(x, 0) = 0; \quad \frac{\partial T_2(x, 0)}{\partial t} = 0; \\ \Delta \xi(0) = 0. \end{aligned} \quad (19)$$

The boundary conditions for  $x = 0$  and  $t > 0$  are

$$T_2(0, t) \neq 0; \quad \frac{\partial T_2(0, t)}{\partial x} = 0; \quad \Delta \xi = \Delta \xi(t) \quad (20)$$

The solution of system of partial differential equations (18) can be found by the well-known methods of operational calculus.

Applying direct and inverse Laplace transformations under the indicated initial and boundary conditions, we obtain an expression for the temperature of the product transformed with respect to the variable  $t$ ,

$$\begin{aligned} T_2(x, p) = \frac{\Psi(p)}{\Phi(p)} \left\{ 1 - \exp \left[ -\frac{\Phi(p)}{v} x \right] \right\} \Delta \xi(p) + \\ + \left\{ \exp \left[ -\frac{\Phi(p)}{v} x \right] \right\} T_2(0, p), \end{aligned} \quad (21)$$

where

$$\Phi(p) = p + \frac{1}{a_2} - \frac{1}{a_2(a_1 p + a_3 + 1)}; \quad (22)$$

$$\Psi(p) = \frac{1}{a_2(a_1 p + a_3 + 1)} \quad (23)$$

The temperature of the product at the heater outlet is

$$\begin{aligned} T_2(L, p) = \frac{\Psi(p)}{\Phi(p)} \left\{ 1 - \exp[-\Phi(p) \tau] \right\} \Delta \xi(p) + \\ + \left\{ \exp[-\Phi(p) \tau] \right\} T_2(0, p). \end{aligned} \quad (24)$$

The variable degree of opening of the regulating valve  $\Delta \xi(p)$  is the regulating parameter; the temperature  $T_2(0, p)$  of the product entering the heater at  $x = 0$  is the perturbation.

The transfer function

$$H_2(p) = \exp[-\Phi(p) \tau] \quad (25)$$

represents the relationship between product temperatures at the intake and outlet of the heater for a constant degree of opening of the regulating valve.

The transfer function

$$H_1(p) = \frac{\Psi(p)}{\Phi(p)} \{ 1 - \exp[-\Phi(p) \tau] \} = \frac{\Psi(p)}{\Phi(p)} [1 - H_2(p)] \quad (26)$$

is the relationship between the temperature of the product at the heater outlet and the regulating parameter  $\Delta \xi$  for  $T_2(0, t) = 0$ .

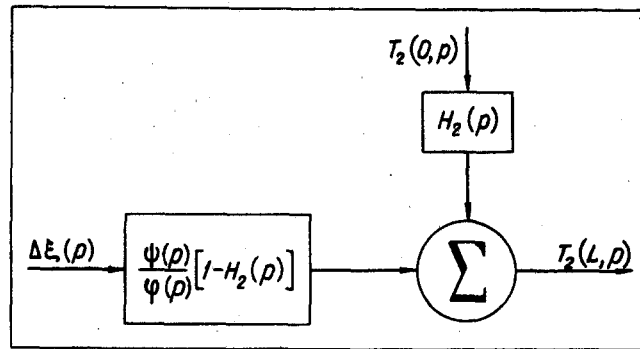


Fig. 1. Block diagram of the heater.

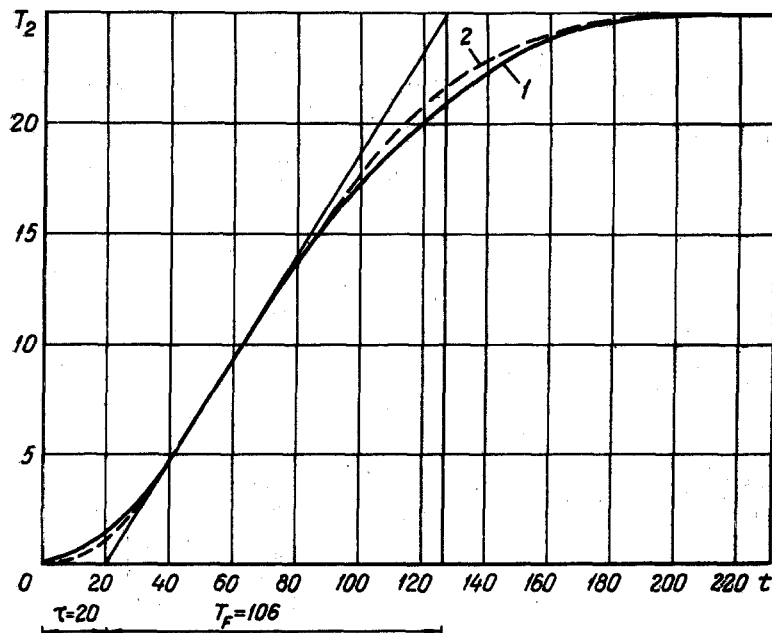


Fig. 2. Excursion characteristic of heater in the regulating "percentage opening of the regulating valve  $\Delta\xi$ -product temperature at outlet  $T_2$ " channel for a perturbation of  $\Delta\xi = 35\%$  opening of the regulating valve.

1) Theoretical characteristic; 2) experimental characteristic.

Hence, the block diagram of a heater with ideal mixing in the case takes the form shown in Fig. 1 if we neglect heat capacity of the wall.

REACTION TO A JUMP PERTURBATION. PERTURBATION IN PRODUCT TEMPERATURE AT THE HEATER INTAKE

If  $T_2(0, t)$  is a step function equal to 0 for  $t < 0$  and to  $T_{20}$  for  $t > 0$ , and if  $\Delta\xi(t) = 0$ , then the dynamic excursion characteristic of the heater with temperature perturbation of the product at the intake can be found by expanding  $H_2(p)$  in an infinite series and subjecting each term of the result to inverse transformation. It is necessary to bear in mind here that since  $T_2(0, t) = T_{20} = \text{const}$  for  $t > 0$ , the transformation of  $T_2(0, p)$  yields the quantity  $T_{20}/p$ .

Then

$$H'_2(p) = \frac{T_2(L, p)}{T_{20}} = \frac{1}{p} H_2(p). \quad (27)$$

Substituting in the value of  $H_2(p)$  from (25) and  $\varphi(p)$  from (22), we obtain

$$H'_2(p) = \frac{1}{p} [\exp(-p\tau)] \times \left[ \exp\left(-\frac{\tau}{a_2}\right) \right] \left[ \exp\frac{\tau}{a_2(a_1p + a_3 + 1)} \right]. \quad (28)$$

Let us expand the last exponential factor in expression (28) in a series and carry out the inverse transformation of Eq. (28),

$$h_2(t) = \exp\left(-\frac{\tau}{a_2}\right) \left\{ \delta(t-\tau) + \left[ \exp\left(-\frac{t-\tau}{a_1(a_3+1)}\right) \times \sum_{n=1}^{\infty} \frac{\left(\frac{\tau}{a_1 a_2}\right)^n (t-\tau)^{n-1}}{n!(n-1)!} \right] u(t-\tau) \right\}, \quad (29)$$

where

$$\delta(t-\tau) = 0 \text{ for } t < \tau; u(t-\tau) = 0 \text{ for } t < \tau;$$

$$\delta(t-\tau) = 1 \text{ for } t \geq \tau; u(t-\tau) = 1 \text{ for } t \geq \tau.$$

Analysis of expression (29) indicates that already for  $t - \tau \geq 1$  sec the term

$$\exp\left[-\frac{t-\tau}{a_1(a_3+1)}\right] \times \sum_{n=1}^{\infty} \frac{\left(\frac{\tau}{a_1 a_2}\right)^n (t-\tau)^{n-1}}{n!(n-1)!} \leq 0.0036.$$

This means that to within a small error we can consider the heater as a delayed-response amplifier link along the "intake product temperature-outlet product temperature" channel.

The heater equation along this channel can be written as

$$T_2(p) = \exp(-p\tau) \exp\left(-\frac{\tau}{a_2}\right) T_{20}. \quad (30)$$

The gain of the amplifier link, in the chosen notation, is given by

$$K_T = \exp\left(-\frac{\tau}{a_2}\right) = \exp\left(-\frac{F_1 k_0}{G_1 c_1}\right). \quad (31)$$

PERTURBATION IN STEAM EXPENDITURE

If  $\Delta\xi(t)$  is a step function (the transformation  $\Delta\xi(p)$  yields  $(1/p)\Delta\xi$ ), i.e., the perturbation in regulating valve opening takes the form of a jump by the amount  $\Delta\xi$  at the instant  $t = 0$ , and if  $T_2(0, p) = 0$ , then the dynamic excursion characteristic of the heater can be found from the following expression for the transfer function (with allowance for (26)):

$$H'_1(p) = \frac{T_2(L, p)}{\Delta\xi} = \frac{1}{p} H_1(p) = \frac{1}{p} \frac{\Psi(p)}{\varphi(p)} [1 - H_2(p)]. \quad (32)$$

Substituting the values of  $\varphi(p)$ ,  $\psi(p)$ , and  $H_2(p)$  from (22) and (28) into (32), expanding the factors of the resulting expression into elementary fractions [2], and carrying out inverse Laplace transformation using the contraction formula, we obtain an expression for the original of the function  $H'_1(p)$ ,

$$h'_1(t) = \frac{a_4}{a_3} \left[ 1 - \frac{\theta_1}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_1}\right) + \frac{\theta_2}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_2}\right) \right] \text{ for } 0 \leq t \leq \tau, \quad (33)$$

$$h'_1(t) = \frac{a_4}{a_3} \left\{ \left[ 1 - \exp\left(-\frac{\tau}{a_2}\right) \right] - \frac{\theta_1}{\theta_1 - \theta_2} \left[ 1 - \exp\frac{\tau(a_2 - \theta_1)}{a_2 \theta_1} \right] \exp\left(-\frac{t}{\theta_1}\right) + \frac{\theta_2}{\theta_1 - \theta_2} \left[ 1 - \exp\left(-\frac{\tau}{a_2} \frac{a_2 - \theta_2}{\theta_2}\right) \right] \times \exp\left(-\frac{t}{\theta_2}\right) \right\} \text{ for } t > \tau, \quad (34)$$

where  $\theta_1$  and  $\theta_2$  are the roots of the equation

$$p^2 + \left(\frac{a_1}{a_3} + \frac{a_2}{a_3} + a_2\right) p^2 + \frac{a_1 a_2}{a_3} = 0. \quad (35)$$

If  $\tau/a_2 \leq 0.22$ , then for  $t > \tau$  we have

$$h'_1(t) \approx \frac{a_4 \tau}{a_2 a_3} \left[ 1 + \frac{a_2 - \theta_1}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_1}\right) - \frac{a_2 - \theta_2}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_2}\right) \right]. \quad (36)$$

Thus, the dynamic excursion characteristic of the heater in the regulating "degree of opening of the regulating valve-product temperature at heater outlet" channel can be expressed as follows:

for  $\tau/a_2 \leq 0.22$ ,

$$T_2(t) = \frac{a_4}{a_3} \left[ 1 - \frac{\theta_1}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_1}\right) + \frac{\theta_2}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_2}\right) \right] \Delta \xi \text{ for } 0 \leq t \leq \tau,$$

$$T_2(t) = \frac{a_4 \tau}{a_3 a_2} \left[ 1 + \frac{a_2 - \theta_1}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_1}\right) - \frac{a_2 - \theta_2}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_2}\right) \right] \Delta \xi \text{ for } t > \tau; \quad (37)$$

for  $\tau/a_2 > 0.22$

$$T_2(t) = \frac{a_4}{a_3} \left[ 1 - \frac{\theta_1}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_1}\right) + \frac{\theta_2}{\theta_1 - \theta_2} \exp\left(-\frac{t}{\theta_2}\right) \right] \Delta \xi \text{ for } 0 \leq t \leq \tau,$$

$$T_2(t) = \frac{a_4}{a_3} \left\{ \left[ 1 - \exp\left(-\frac{\tau}{a_2}\right) \right] - \frac{\theta_1}{\theta_1 - \theta_2} \left[ 1 - \exp\left(\frac{\tau(a_2 - \theta_1)}{a_2 \theta_1}\right) \right] \exp\left(-\frac{t}{\theta_1}\right) + \frac{\theta_2}{\theta_1 - \theta_2} \left[ 1 - \exp\left(\frac{\tau(a_2 - \theta_2)}{a_2 \theta_2}\right) \right] \times \right. \\ \left. \times \exp\left(-\frac{t}{\theta_2}\right) \right\} \Delta \xi \text{ for } t > \tau. \quad (38)$$

To determine the gain, time constant, and lag in the regulating channel of the heater we must construct the excursion characteristic described by Eqs. (37) or (38).

#### SAMPLE CALCULATION OF THE DYNAMIC CHARACTERISTICS OF A CASE-TUBE HEATER

Initial data for the Lang three-step tomato pulp heater: eleven tubes in product conduit cluster; outside diameter of tubes 0.055 m; tube wall thickness 0.0025 m; tube length  $l = 3.5$  m; outside diameter of case 0.5 m; case wall thickness 0.008 mm;  $P_m = 49 \cdot 10^4$  N/m<sup>2</sup>;  $P_{10} = 7.845 \cdot 10^4$  N/m<sup>2</sup>; volume feed rate of product through heater  $v_2 = 3.33 \cdot 10^{-3}$  m<sup>3</sup>/sec;  $k_0 = 291$  J/°K · m<sup>2</sup> · sec;  $i'' = 2.7 \cdot 10^6$  J/kg;  $T_{10} = i_0' = 488 \cdot 10^3$  J/kg;  $c_2 = 0.48 \cdot 10^3$  J/kg · °K;  $c_1 = 4.02 \cdot 10^3$  J/kg · °K;  $\gamma_2 = 1021.4$  kg/m<sup>3</sup>; solid content of pulp 5%;  $C = 32$ ;  $\varepsilon_{ef} = 0.736$ ;  $\xi_0 = 0.5$ ;

a) from the table for dry saturated steam and from (4) for  $P_{10} = 7.845 \cdot 10^4$  N/m<sup>2</sup> we obtain  $\gamma_{1m} = 1.005$  kg/m<sup>3</sup>;  $\alpha = 0.0316$  kg/m<sup>3</sup> · °K;  $U_V = 2.54 \cdot 10^6$  J/kg;  $\beta = 1.1 \cdot 10^3$  J/kg · °K;  $\eta = 6.13 \cdot 10^3$  N/m<sup>2</sup> · °K;

b) on the basis of the geometric dimensions of the heater we find that  $V_1 = 0.349$  m<sup>3</sup>;  $C = 153.32 \cdot 10^3$  J/kg;  $F_1 = 18.85$  m<sup>2</sup>;  $f = 216 \cdot 10^{-4}$  m<sup>2</sup>;

c) from expression (6) we obtain the value  $\gamma = 175.8 \cdot 10^3$  J/°K;

d) from expression (15) we obtain  $C_1 = 875 \cdot 10^3$  J/°K;

e) from (17a) and (10a) we obtain  $a_1 = 32.1$  sec,  $a_2 = 160$  sec;

f) to determine the coefficients  $a_3$  and  $a_4$  we first obtain the following values from (7) and (10):  $G_{10} =$

$= 0.2125$  kg/sec;  $\eta_0 = 2.213 \cdot 10^6$  J/kg;  $a = 0.426$  kg/sec · % opening of the regulating valve;  $a' = 1.6 \cdot 10^{-3}$  kg/sec · °K. We then find from (17b) that  $a_3 = 0.795$ ,  $a_4 = 1.72$  °K/% opening of the regulating valve;

g)  $\tau = 3lf/v_2 = 64.3$  sec;

h) heater gain in the perturbing "product temperature at intake-product temperature at outlet" channel is given by (31)

$$K_T = \exp\left(-\frac{\tau}{a_2}\right) = 0.67;$$

i) the heater equation in the perturbing channel is

$$T_2(p) = \exp(-p \cdot 64.3) 0.67 T_{20};$$

j) since  $\tau/a_2 > 0.22$ , it follows that in order to determine the dynamic properties of the regulating channel of the heater it is necessary to construct the excursion characteristic described by Eq. (38).

This characteristic is shown in Fig. 2.

From the excursion characteristics we find that  $\tau = 20$  sec,  $T_F = 106$  sec,  $K_F = \frac{a_4}{a_3} \times [1 - \exp(-\frac{\tau_0}{a_2})] = 0.71$  °K/% opening of the regulating valve.

The dashed curve represents the experimental excursion curve.

#### NOTATION

$C$  is the specific carrying capacity of the regulating valve at its maximum opening;  $C_1$  is the specific heat of the product, J/°K;  $C_3$  is the specific heat of the outer surface of the case, J/°K;  $F_1$  is the heat exchange surface between the steam and product, m<sup>2</sup>;  $G_1$  is the variable steam feed rate by weight relative to the nominal feed rate  $G_{10}$ , kg/sec;  $G_2$  is the variable condensate feed rate relative to the nominal feed rate  $G_{20}$ , kg/sec;  $L$  is the over-all length of the product conduit, m;  $P_1$  is the steam pressure in the heater, N/m<sup>2</sup>;  $P_{10}$  is the average steam pressure in the heater, N/m<sup>2</sup>;  $P_m$  is the steam pressure in the main, N/m<sup>2</sup>;  $Q_1$  is the thermal loss, J/sec;  $T_1$  is the variable steam pressure, °K;  $T_2$  is the variable product temperature, °K;  $U_V$  is the internal energy of the steam, J/kg;  $U_m$  is the internal energy of the steam under average operating conditions, J/kg;  $V_1$  is the volume of the steam space, m<sup>3</sup>;  $c_1$  is the specific heat of the product, J/kg · °K;  $c_c$  is the specific heat of the condensate ( $c_c = 4.19 \cdot 10^3$  J/kg · °K);  $f$  is the cross sectional area of the product stream, m<sup>2</sup>;  $i_0'$  is the enthalpy of the condensate under average operating conditions, J/kg, which is numerically equal to the initial steam temperature  $T_{10}$ ;  $i''$  is the enthalpy of the steam entering the heater, J/kg;  $k_0$  is the over-all heat transfer coefficient, J/m<sup>2</sup> · sec · °K;  $p$  is the differentiation operator;  $t$  is the time, sec;  $v$  is the velocity of the product, m/sec;  $\alpha$  is a coefficient allowing for the dependence of steam density on temperature, kg/m<sup>3</sup> · °K;  $\beta$  is a coefficient allowing for the dependence of the internal energy of the steam on temperature, J/kg · °K;  $\gamma$  is the effective specific heat of the steam space, J/°K;  $\gamma_{10}$  is the specific weight of the steam, kg/m<sup>3</sup>;  $\gamma_{1m}$  is the specific weight of the steam under average operating conditions, kg/m<sup>3</sup>;  $\gamma_2$  is the specific weight of the product, kg/m<sup>3</sup>;  $\varepsilon_{ef}$  is a correction factor which allows for the effect of expansion;  $\xi$  the degree of opening of the regulating valve, expressed as the % opening of the regulating valve;  $\eta$  is a coefficient allowing for the dependence of the pressure of the saturated steam on its temperature, N/m<sup>2</sup> · °K;  $\tau = L/v$  is the total time of travel of the product through the heater, sec.

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All-Union Institute of Food  
Industry Automation, Odessa